[This question paper contains 6 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1564 A

Unique Paper Code : 42351201

Name of the Paper : Calculus and Geometry, CBCS (LOCF)

Name of the Course : B.Sc. (Programme) Mathematical Sciences / Physical Sciences

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Marks are indicated.

1. (a) Sketch the graph of the function $f(x) = x^4 + 2x^3$. (6.5)

P.T.O.

(b) Sketch a graph of a function f with all the following properties :

The graph has $y = 1$ and $x = 3$ as asymptotes; f is increasing for $x < 3$ and $3 < x < 5$ and is decreasing elsewhere; the graph is concave up for $x < 3$ and for $x > 7$ and concave down for $3 < x < 7$; $f(0) = 4 = f(5)$ and $f(7) = 2$. (6.5)

(c) Identify the symmetries of the curve $r = 1 + 2\cos\theta$ and then sketch the curve. (6.5)

(d) Let $x = 4\tan 2t$, $y = 3\sec 2t$ where $0 \leq t \leq \pi$. Find an explicit relation between x and y. Also, sketch the path described by the given parametric equations over the prescribed interval. (6.5)

2. (a) Evaluate the following limits using L'Hospital's Rule

$\lim_{x\to\pi/2}(\tan x)^{(\pi/2)-x}$ and $\lim_{x\to0^+}[\tan x \log x]$. (6)

(b) Sketch the graph of $y = (x - 4)^{2/3}$. (6)

(c) Use cylindrical shells to find the volume of the solid that is generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the line $x = 0$. (6)

(d) Find the volume of the solid formed when the region between the graphs of $y = 1 + 2x^2$ and $y = 3 - 2x^2$ is revolved about the x-axis. (6)

3. (a) Find the volume of the solid generated when the region enclosed by $x = 0$, $y = 0$, $x = 1$ and $y = x^2 + 1$ is revolved about the y-axis. (6.5)

(b) Find the length of the curve $y = 2x^2 + 1$ over the interval [1,3]. (6.5)

(c) Find the area of the surface swept out by revolving $y = \sqrt{9 - x^2}$ about the x-axis. (6.5)

(d) Find the arc length of the curve $x = t^3$, $y = 3\frac{t^2}{2}$, $0 \le t \le \sqrt{3}$. (6.5)

4. (a) Derive the reduction formula

$$\int \sin^n x\, dx = -\frac{1}{n}\sin^{n-1}x \cos x + \frac{n-1}{n}\int \sin^{n-2}x\, dx$$

and evaluate the integral $\int_0^{\pi/2} \sin^5 x\, dx$. (6)

P.T.O.

(b) Prove that for nonnegative integers m and n,

$$\int_0^{2\pi} \cos mx \cos nx \, dx = 0.$$

(6)

(c) Obtain reduction formula

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx, \quad n > 2$$

and hence using that evaluate the integral $\int \tan^4 x \, dx$.

(6)

(d) Evaluate $\int_0^{\pi/2} \sin^2 \frac{x}{2} \cos^m \frac{x}{2} dx$ after obtaining the reduction formula for the integral

$$\int \sin^m x \cos^n x \, dx.$$

(6)

5. (a) Describe the graph of the equation

$$y = 4x^2 + 8x + 5.$$

(6.5)

(b) Find equation for the hyperbola that has vertices $(0, \pm 2)$ and asymptotes

$$y = \pm \frac{2}{3} x.$$

(6.5)

(c) Sketch the graph of ellipse and label the foci, vertices and ends of the minor axis

$(x + 3)^2 + 4(y - 5)^2 = 16.$ (6.5)

(d) Rotate the axes of coordinates to get rid of the xy-term from the equation, name the conic $x^2 + 4xy - 2y^2 - 6 = 0$ and sketch its graph. (6.5)

6. (a) Show that the graphs of given $r_1(t)$ and $r_2(t)$ intersect at the point P(1, 1, 3). Find the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at this point, where

$$r_1(t) = t^2\hat{i} + t\hat{j} + 3t^3\hat{k}$$

$$r_2(t) = (t-1)\hat{i} + \frac{1}{4}t^2\hat{j} + (5-t)\hat{k}.$$ (6)

(b) Sketch the graph and show direction of increasing t for

$$r(t) = 9\cos t\,\hat{i} + 4\sin t\,\hat{j} + t\hat{k}.$$ (6)

(c) Evaluate $\nabla \times (\nabla U \times \nabla V)$ where $U = x^2yz$, $V = xy - 3z^2$. (6)

P.T.O.

(d) Show that divergence of the field

$$F(x, y, z) = \frac{c}{\left(x^2 + y^2 + z^2\right)^{3/2}}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \text{ is zero.}$$

(6)